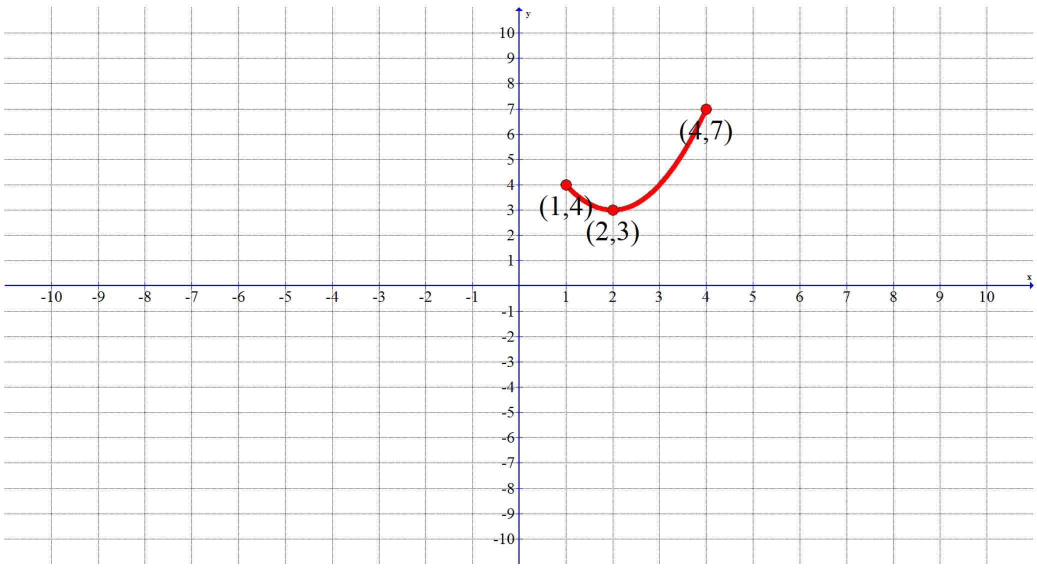
Section 4.1 Absolute Extrema

#1-9: Find the absolute maximum and absolute minimum

1)



(4,7) is the highest point. Its y coordinate is the absolute maximum.

(2,3) is the lowest point. It’s y-coordinate is the absolute minimum.

(1,4) is neither the highest nor the lowest point. It is not an absolute maximum, nor an absolute minimum.

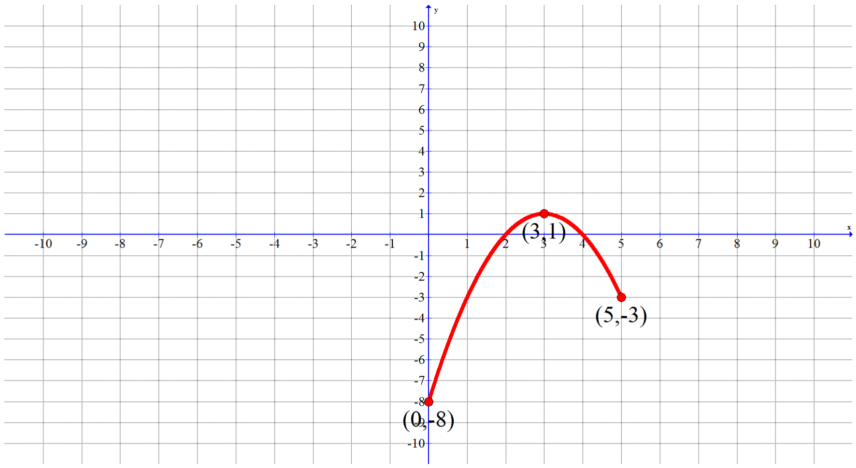
It would be best to write your answer as follows:

Solution:

There is an absolute maximum of y = 7, which occurs when x = 4.

There is an absolute minimum of y = 3, which occurs when x = 2.

3)



(3,1) is the highest point. Its y coordinate is the absolute maximum.

(0,-8) is the lowest point. It's y-coordinate is the absolute minimum.

(5,-3) is neither the highest nor the lowest point. It is not an absolute maximum, nor an absolute minimum.

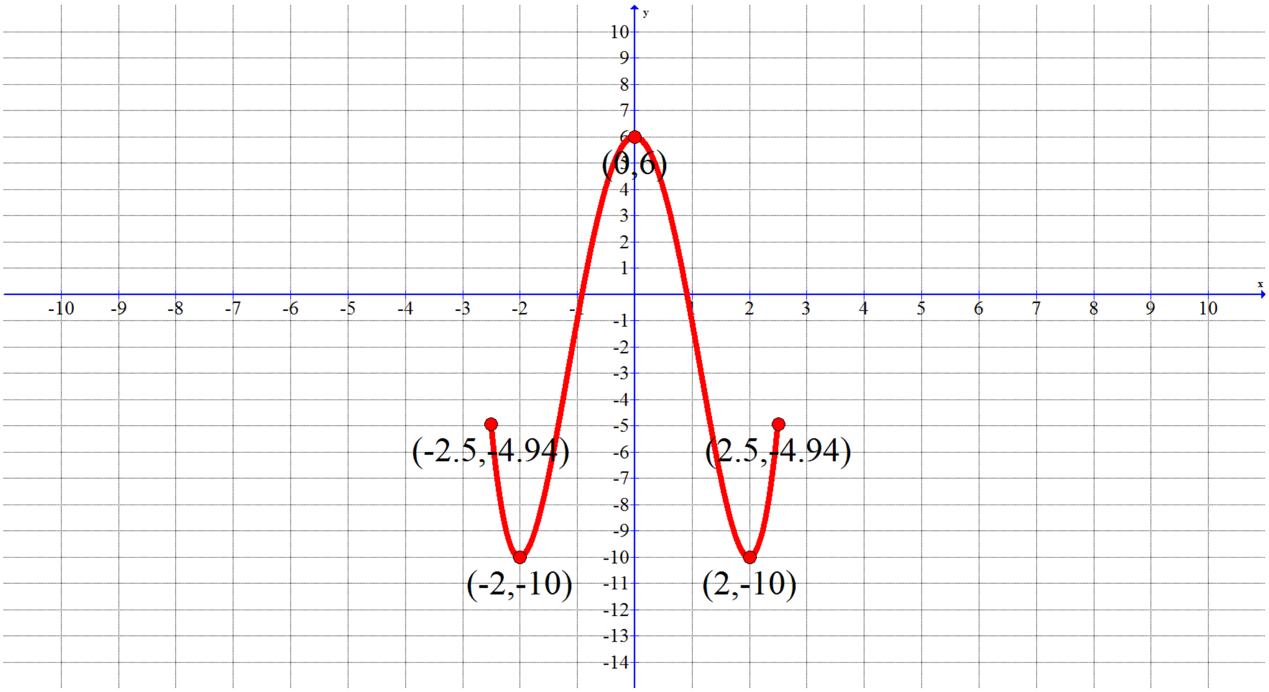
It would be best to write your answer as follows:

Solution:

There is an absolute maximum of y = 1, which occurs when x = 3.

There is an absolute minimum of y = -8, which occurs when x = 0.

5)



(0,6) is the highest point. Its y coordinate is the absolute maximum.

(-2,-10) and (2,-10) are tied for the lowest point. Their y-coordinate is the absolute minimum.

(-2.5, -4.94) and (2.5, -4.94) are neither the highest nor the lowest point. They are not an absolute maximum, nor an absolute minimum.

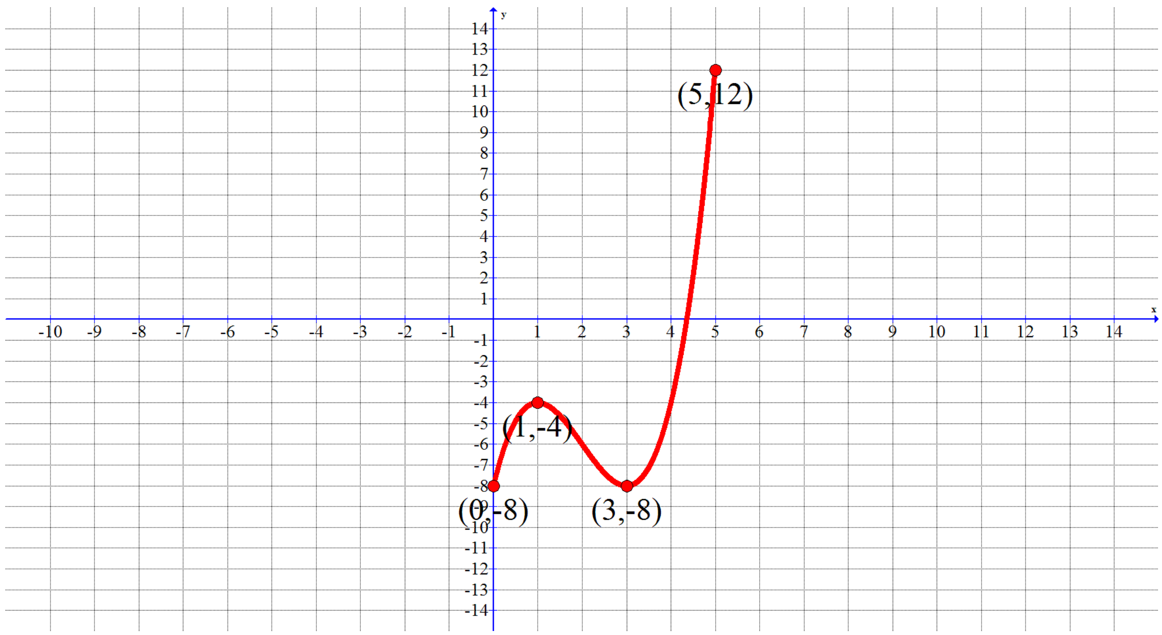
It would be best to write your answer as follows:

Solution:

There is an absolute maximum of y = 6, which occurs when x = 0.

There is an absolute minimum of y = -10, which occurs when x = -2 and x = 2.

7)



(5,12) is the highest point. Its y coordinate is the absolute maximum.

(0,-8) and (3,-8) are tied for the lowest point. Their y-coordinate is the absolute minimum.

(1,-4) is not a point to consider as it is neither the highest nor lowest point.

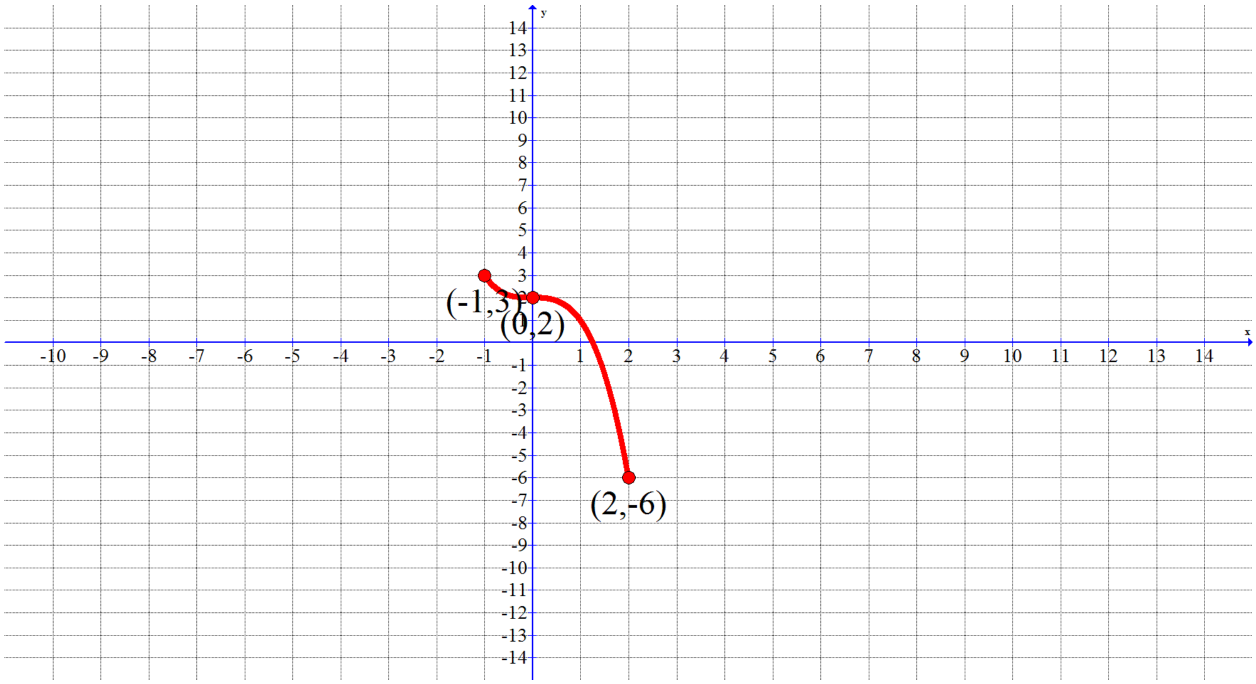
It would be best to write your answer as follows:

Solution:

There is an absolute maximum of y = 1, which occurs when x = 5.

There is an absolute minimum of y = -8, which occurs when x = 0 and x = 3.

9)



(-1,3) is the highest point. It's y coordinate is the absolute maximum.

(2,-6) is the lowest point. It's y-coordinate is the absolute minimum.

(0,2) is neither the highest nor the lowest point. It is not an absolute maximum, nor an absolute minimum.

It would be best to write your answer as follows:

Solution:

There is an absolute maximum of y = 3, which occurs when x = -1.

There is an absolute minimum of y = -6, which occurs when x = 2.

#10-27: Find the absolute maximum and absolute minimum of the function under the given interval.

11) $f(x) = x^2 - 6x + 4; \quad [-5,5]$

First find all values of x between -5 and 5 where f'(x) = 0

f'(x) = 2x – 6

2x – 6 = 0

2x = 6

x = 3 (is the value of x between -5 and 5 where f'(x) = 0)

Next make a table of values, place -5,5 and 3 in the x-column. Then find the corresponding y-value by plugging the x-values into the **original function**.

X - value	Y-value	Point on graph
-5	$f(-5) = (-5)^2 - 6(-5) + 4 = 59$	(-5, 59) abs max
5	$f(5) = (5)^2 - 6(5) + 4 = -1$	(5, -1)
3	$f(3) = (3)^2 - 6(3) + 4 = -5$	(3, -5) abs min

Answer:

absolute max y = 59, when x = -5

absolute min y = -5, when x = 3

13) $f(x) = x^3 + 6x^2; \;\; [-2,1]$

First find all values of x between -2 and 1 where f'(x) = 0

$f'(x) = 3x^2 + 12x$

$3x^2 + 12x = 0$

$3x(x+4) = 0$

$3x = 0, \qquad x + 4 = 0$

$x = 0/3, \qquad x = -4$ (ignore this as it is not between -2 and 1)

$x = 0$

Next make a table of values, place -2,1 and 0 in the x-column. Then find the corresponding y-value by plugging the x-values into the **original function**

X - value	Y-value	Point on graph
-2	$f(-2) = (-2)^3+6(-2)^2= 16$	(-2, 16) abs max
1	$f(1) = (1)^3+6(1)^2= 7$	(5, -1)
0	$f(-2) = (0)^3+6(0)^2= 0$	(0, 0) abs min

Answer:

absolute max y = 16, when x = -2

absolute min y = 0, when x = 0

15) $f(x) = x^3 - 3x^2 + 2;\ \ [-1,5]$

First find all values of x between -1 and 5 where $f'(x) = 0$

$f'(x) = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$3x = 0$ $\qquad\qquad$ $x - 2 = 0$

$x = 0$ $\qquad\qquad$ $x = 2$

Next make a table of values, place -1,5, 2 and 0 in the x-column. Then find the corresponding y-value by plugging the x-values into the **original function**

X - value	Y-value	Point on graph
-1	$f(-1) = (-1)^3 - 3(-1)^2 + 2 = -2$	(-1, -2) abs min
5	$f(5) = (5)^3 - 3(5)^2 + 2 = 52$	(5, 52) abs max
2	$f(2) = (2)^3 - 3(2)^2 + 2 = -2$	(2, -2) abs min
0	$f(0) = (0)^3 - 3(0)^2 + 2 = 2$	

Answer:

absolute max y = 52, when x = 5

absolute min y = -2, when x = -1 and x = 2

17) $f(x) = 3x^4 - 4x^3$; [-2,3]

First find all values of x between -2 and 3 where $f'(x) = 0$

$f'(x) = 12x^3 - 12x^2$

$12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$

$12x^2 = 0$ $\quad\quad$ $x - 1 = 0$

$x^2 = 0$ $\quad\quad$ $x = 1$

$x = 0$

Next make a table of values, place -2,3, 1 and 0 in the x-column. Then find the corresponding y-value by plugging the x-values into the **original function**

X - value	Y-value	Point on graph
-2	$f(-2) = 3(-2)^4 - 4(-2)^3 = 80$	
3	$f(3) = 3(3)^4 - 4(3)^3 = 135$	(3, 135) abs max
1	$f(1) = 3(1)^4 - 4(1)^3 = -1$	(1, -1) abs min
0	$f(0) = 3(0)^4 - 4(0)^3 = 0$	

Answer:

absolute max y = 135, when x = 3

absolute min y = -1, when x = 1

19) $f(x) = (x^2 - 16)^3$; [-2,2]

First find all values of x between -2 and 2 where $f'(x) = 0$

$f'(x) = 3(2x)(x^2 - 16)^2$

$f'(x) = 6x(x^2 - 16)^2$

$6x(x^2 - 16)^2 = 0$

$6x = 0$ $(x^2 - 16)^2 = 0$

$x = 0$ enough to just solve $x^2 - 16 = 0$

$(x+4)(x-4) = 0$

$x = -4$, $x = 4$ (ignore both)

Next make a table of values, place -2,2 and 0 in the x-column. Then find the corresponding y-value by plugging the x-values into the **original function**

X - value	Y-value	Point on graph
-2	$f(-2) = ((-2)^2 - 16)^3 = -1728$	Abs max (-2, -1728)
2	$f(2) = ((2)^2 - 16)^3 = -1728$	Abs max (2, -1728)
0	$f(0) = ((0)^2 - 16)^3 = -4096$	Abs min (0, - 4096)

Answer:

absolute max y = -1728, when $x = \pm 2$

absolute min y = -4096, when x = 0

21) $f(x) = \sqrt[5]{x}; \quad [-3,2]$

First find all values of x between -3 and 2 where f'(x) = 0, or where f'(x) is undefined.

Rewrite with a fraction exponent to make it easier to find f'(x)

f(x) = $x^{1/5}$

$$f'(x) = \frac{1}{5}x^{1/5 - 5/5} = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$$

$$f'(x) = \frac{1}{5\sqrt[5]{x^4}}$$

f'(x) = 0 when the numerator equals 0. That is f'(x) = 0 when 1 = 0. Since there is no x, there is no value of x where f'(x) = 0

21 continued

f’(x) is undefined when its denominator equals 0. To do this set the denominator of the derivative equal to 0.

$$5\sqrt[5]{x^4} = 0$$

First divide both sides by 5

$$\sqrt[5]{x^4} = 0$$

Then raise both sides to the 5^{th} power to cancel the radical.

$$\left(\sqrt[5]{x^4}\right)^5 = 0^5$$

This gives

$x^4 = 0$

Now take the 4^{th} root of both sides. I would need a plus or minus sign if the 0 were some positive or negative number.

$$\sqrt[4]{x^4} = \sqrt[4]{0}$$

x = 0 (this is a number that needs to go in my table)

21) concluded

Next make a table of values, place -3,2 and 0 in the x-column. Then find the corresponding y-value by plugging the x-values into the **original function**

X - value	Y-value	Point on graph
-3	f(-3) $= \sqrt[5]{-3} = -1.25$	Abs min (-3, $\sqrt[5]{-3}$)
2	f(2) = $\sqrt[5]{2} = 1.15$	Abs max (2, $\sqrt[5]{2}$)
0	f(0) =0	

Answer:

absolute min y = $\sqrt[5]{-3}$, when x = -3

absolute max y = $\sqrt[5]{2}$, when x = 2

23) $f(x) = 2xe^{x}$;   [0,3]

$f'(x) = 2xe^x + 2e^x$

$f'(x) = 2e^x(x + 1)$

$2e^x = 0$ $x + 1 = 0$

No solution $x = -1$ ignore this as it is not in the

Interval [0,3]

Only test the endpoints as the critical number is not in the interval.

x	f(x)	point
0	$f(0) = 2(0)e^0 = 0$	(0,0) abs min
3	$f(3) = 2(3)e^3 = 6e^3 = 120.51$	$(3,6e^3)$ abs max

Abs max of $y = 6e^3$ when $x = 3$

Abs min of $y = 0$ when $x = 0$

25) $f(x) = e^{x^2}$; $[-2,1]$

f'(x) = $2xe^{x^2}$

$2xe^{x^2} = 0$

2x = 0 $\qquad$ $e^{x^2} = 0$

x = 0 $\qquad$ no solution

x	f(x)	point
0	f(0) = $e^{0^2} = 1$	(0,1) abs min
-2	f(-2) = $e^{(-2)^2} = e^4$ =54.60	(-2,e^4) abs max
1	f(1) = $e^1 = e$ =2.72	

Abs max y = e^4 when x = -2

Abs min y = 1 when x = 0

27) $f(x) = x^2 e^x$; [-3,1]

$f'(x) = 2xe^x + x^2e^x$

$f'(x) = xe^x(2 + x)$

$xe^x(2+x) = 0$

$x = 0$	$e^x = 0$	$2 + x = 0$
$x = 0$	no sol	$x = -2$

x	f(x)	point
-3	$f(-3) = (-3)^2e^{-3} = 9/e^3 = .45$	$(-3, 9/e^3)$
1	$f(1) = (1)^2e^1 = e = 2.72$	(1,e) abs max
0	$f(0) = (0)^2e^0 = 0$	(0,0) abs min
-2	$f(-2) = (-2)^2e^{-2} = 4/e^2 = .54$	$(-2, 4/e^2)$

Abs max y = e when x = 1

Abs min y = 0 when x = 0